STATISTICAL ESTIMATION OF SOME METEOROLOGICAL VARIABLES USING THE BETA KERNEL FUNCTION

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Abstract

The study of meteorological variables is imperative due to their usefulness in our daily activities. Climate change which is now a global issue has received considerable large volume of research owing to the fact that its effects cut across all aspects of life. It is generally known that the effects of climatic change pose a great threat to agricultural activities in Africa continent and the world at large. Temperature and relative humidity are inversely related and this inverse relationship as investigated using real life data shows its devastating effect on the environment if neglected without urgent control measure. Hence, this study investigates the relationship between temperature and relative humidity using the beta kernel function with the asymptotic mean integrated squared error (AMISE) as the criterion function. The results of the study using real data reveals that temperature and relative humidity greatly determine climatic fluctuations which can adversely affect the environment.

Key Words: *Bandwidth, Climate change, Density, Humidity, Kernel, Temperature*

Introduction

Human activities are dependent on all the weather variables. Agricultural activities rely largely on the relationship between some climatic variables for quality or maximum yield. Due to the importance of agricultural products to mankind, efforts are being made globally to curb the activities that could result in climate change (Smith, 2000). A study of the effects of climate change was conducted on Near East North Africa (NENA) where the agricultural activities and livelihoods of people within the countries in the region were critically examined with emphasis on small-scale farming by Food and Agriculture Organization of the United Nations in Cairo, Egypt (Lewis *et al*., 2018; Nwankwo and Ukhurebor, 2019).

Apart from the direct effects of climate change on agricultural products, the mobility of the products and human resources to required markets can also be hampered by climatic fluctuations. The interactions between temperature and

humidity were investigated in relation with cloud cover using aircraft variables to verify if temperature and humidity changes are correlated and the results revealed negative correlations between the variables (Tompkins, 2003). Globally, the effects of climate change can affect access to food which directly affects the Sustainable Development Goals (SDGs) of poverty and hunger eradication. A large quantity of food is transported via waterway across the globe, during which there are changes in the weather and this could directly affect the waterway, invariably cause difficulties in transporting such products if there are no alternatives routes, and if there are transportation problems, the volume of products to be exported to the international market will be affected hence; the global prices for food will also be affected drastically (Brown *et al*., 2015).

Climatic fluctuations are not just a threat to the growth and development of the economic and agricultural sector of nations but its effects can be felt in the entire human population. The human race now faces new climatic fluctuations at an exponential rate in their environment in which adaptability has not been fully developed (Adejuwon, 2004). The spread of diseases in animal and humans can also be attributed to changes in climatic variables; hence adequate measures should be taken to address this menace. One of the important climatic variables is rainfall and insufficient amount of rainfall could result in degeneration and deforestation of land as a resource leading to desertification. Due to the adverse effects of climatic fluctuations on human race and its environment, it is regarded as the severest challenge confronting the world now and its effects is worse than the

activities of terrorists globally (Niggol and Mendelson, 2008).

Climatic variables should be properly investigated so that inherent structures can be identified and decisions can be made on reducing its effects on the environment. Orderliness of climatic variables is fundamental to curbing its activities; hence statistical analysis of some climatic data is the focus of this work. Data analysis and estimation has broad spectral in applicability virtually in all fields of studies. Analysis of data could be for visualization, exploratory and estimation purposes for communication of the findings about the observations. The visualization of data involves representation of the data graphically in order to identify trends and patterns that are present in the data. In exploratory analysis, the data are subjected to critical examination with the aid of some statistical tools while the data estimation stage uses the actual data values in predicting or forecasting a future occurrence (Bobadoye *et al*., 2020).

Accurate predictions of climatic variables with the application of statistical tools could prevent disasters that are connected to climate change. Predictions or forecasting using statistical tools involves density estimation which is the construction of probability estimates using the data either from a known probability distribution or unknown distribution. If the observations are assumed to be members of a known probability distribution, then the estimation technique is parametric density estimation. In this estimation method, prior knowledge of the data is required and it is only the parameter of the distribution that will be estimated. On the other hand, nonparametric approach does not make any assumption about the

distribution of the set of observations but the observations are subjected to self-examination. Nonparametric estimation gives a better approach to statistical analysis of data due to its ability to capture the structures of the underlying distribution. One of the features of nonparametric estimation is that they are useful exploratory and visualization tools in statistical analysis. This advantages account for nonparametric models as a better choice of robust and accurate statistical tools (Hansen, 2019). Nonparametric density estimation is of wide applicability in multivariate analysis of data for visualization and exploratory purposes especially for observations whose prior information may not be known (Siloko *et al.*, 2019a). Due to constant variations in weather variables, the nonparametric approach will be employed to avoid the assumption of imposition of distributional property on the observations.

This paper focused on density estimation with emphasis on weather variables using the beta kernel functions for accurate meteorological information. The analysis is based on bivariate kernels only since it considers two weather variables. The general form of the kernel estimator is presented, and the beta polynomial kernel functions are discussed. The smoothing parameters and kernel estimates of the bivariate product kernel using real data were obtained and the kernel performances were evaluated with the asymptotic mean integrated squared error. The aim of this study is to investigate the relationship between temperature and relative humidity using the beta kernel function and their effects on the environment.

Mathematical Formulation

The kernel density estimator is a probability density function with applications in exploratory data analysis and data visualization (Rosenblatt, 1956; Parzen, 1962). It is also of indirect application in discriminant analysis, goodness-of-fit testing, hazard rate estimation and other statistical related estimation techniques (Raykar *et al.*, 2015). The prominence of kernel estimators in nonparametric density estimation is attributed to their simplicity and computational efficiency. As a standardized weighting function, the univariate kernel density estimator is

$$\hat{f}(\text{x}) = \frac{1}{nh_\text{x}} \sum_{i=1}^{n} K\left(\frac{\text{x} - X_i}{h_\text{x}}\right), \tag{1}$$

where $K(\cdot)$ is a kernel function, n is sample size, $h_\text{x} > 0$ is bandwidth (also called smoothing parameter), x is range of the observations and X_i is the set of observations. Usually, the kernel function is a non-negative function that must satisfy the following conditions

$$\int K(\text{x})dx = 1, \int \text{x}K(\text{x})dx = 0 \text{ and } \int \text{x}^2 K(\text{x})dx \neq 0. \tag{2}$$

The implication of the first condition in Equation (2) in kernel method is that every kernel function must integrate to unity that is one; hence most kernel functions are probability density functions. The other conditions imply that the average of the kernel is zero while the variance of the kernel $\mu_2(K)$ is not zero (Siloko *et al.*, 2020a; Scott, 2015).

Density estimation using the kernel estimator involves the smoothing parameter which plays a significant role in the estimation process; hence it is regarded as a resolution factor when viewing observations. The evaluation of

the kernel estimator is strictly dependent on the smoothing parameter; therefore, its appropriate choices are very imperative. Smoothing parameter selection is the major problem confronting the implementation of kernel density estimation. There are plethora smoothing parameter selectors but no universally acceptable selector for all situations; hence, smoothing parameter selections is a grey area in kernel density estimation. The problem of smoothing parameter selection in univariate kernel is not as complex as in the multivariate kernel estimation with different forms of parameterizations (Siloko *et al*., 2018). The general multivariate form of the kernel estimator in Equation (1) with a single bandwidth is of the form

$$\hat{f}(x) = \frac{1}{nh_x^d} \sum_{i=1}^{n} K\left(\frac{x - X_i}{h_x}\right), \qquad (3)$$

where d is dimension of the kernel function. The multi-dimensional kernel also satisfies the conditions in Equation (2) and with the assumption of its contours being spherically symmetric. This multivariate kernel estimator is advantageous because the formulas for the asymptotic mean integrated squared error and the optimal smoothing parameter value can be obtained unlike other complex forms of parameterizations without explicit optimal bandwidth formula.

The two-dimensional kernel estimators can be easily obtained from the general form using the product techniques that applies different smoothing parameter for each dimension (Sain *et al*., 1994). The bivariate product kernel uses the product of two univariate kernel but with different level of smoothing in each axis. In bivariate kernel density estimation, x, y are random variables having joint density function $f(x, y)$, with $X_i, Y_i, i = 1, 2, …, n$ as set of observations, where n is the sample size. The bivariate product kernel density estimator is of the form

$$\hat{f}(x, y) = \frac{1}{nh_x h_y} \sum_{i=1}^{n} K\left(\frac{x - X_i}{h_x}, \frac{y - Y_i}{h_y}\right) = \frac{1}{n\, h_x h_y} \sum_{i=1}^{n} K\left(\frac{x - X_i}{h_x}\right) K\left(\frac{y - Y_i}{h_y}\right), \qquad (4a)$$

where $h_x > 0$ and $h_y > 0$ are smoothing parameters in X and Y directions, x and y are ranges of the variables in the different axes and $K(x, y)$ is a bivariate kernel function, which is the product of two univariate kernels. The bivariate product kernel is mostly beneficial if there are variations in the scales of the observations in the respective axes or directions. The bivariate kernel bridges the gap between the univariate and other higher dimensional kernel estimators. An advantage of the bivariate kernel estimators is that their estimates are simple to understand and interpret, either as surface plots or contour plots and also a useful tool for data exploratory analysis and data visualization (Silverman, 2018). The general form of the multivariate product kernel is

$$\hat{f}(x) = n^{-1} \left(\prod_{j=1}^{d} h_j\right)^{-1} \sum_{i=1}^{n} K\left(\frac{x_1 - X_{i1}}{h_1}, \frac{x_2 - X_{i2}}{h_2}, …, \frac{x_d - X_{id}}{h_d}\right), \qquad (4b)$$

where h_j are the smoothing parameter for the different axes and d is the dimension of the kernel.

The Beta Polynomial Kernel Functions

There is plethora of kernel estimators, however; the beta polynomial kernel whose degree of differentiability is dependent on the power of the polynomial is our focus because its limiting case is the popular Gaussian kernel that has wide applications. The beta polynomial kernels are probability density function with their general form given by

$$K_p(t) = \frac{(2p+1)!}{2^{2p+1}(p!)^2}(1-t^2)^p, \qquad (5)$$

where $p = 0, 1, 2, \dots, \infty$ is regarded as its power and t takes value within the interval $-1 \le t \le 1$. Since the beta kernel functions are probability density functions, they are usually evaluated within this interval $[-1, \ 1]$. The values of p that ranges from 0 to 3 will produce Uniform, Epanechnikov, Biweight and Triweight kernels with the uniform kernel being the simplest kernel, while at its limiting case the resulting kernel is the Gaussian kernel also known as the normal kernel. The limiting case simply means when the value of p tends to infinity (Siloko *et al.*, 2019b). The corresponding kernel functions for $p = 1$, 2, and 3 known as the Epanechnikov, Biweight and Triweight kernel are as follows.

$$K_1(t) = \frac{3}{4}(1-t^2) \qquad (6)$$

$$K_2(t) = \frac{15}{16}(1-t^2)^2 \qquad (7)$$

$$K_3(t) = \frac{35}{32}(1-t^2)^3 \qquad (8)$$

As the value of p tends to infinity, the resulting kernel is the popular Gaussian kernel given by

$$K_\emptyset(t) = \frac{1}{\sqrt{2\pi}} exp\left(-\frac{t^2}{2}\right). \qquad (9)$$

The Epanechnikov, Biweight, and Triweight kernel functions are of wider applications especially, the Epanechnikov kernel which is the optimal kernel that is used in computation of the efficiency of other kernel functions of the family

In the product method, several univariate kernels are multiplied to obtain the corresponding multivariate kernel and are of the form

$$K_p(t) = A^2 \prod_{i=1}^{2}(1-t_i^2)^p, \qquad (10)$$

where $A = \frac{(2p+1)!}{2^{2p+1}(p!)^2}$ is the normalization constant of the kernel function. The bivariate beta kernel functions can simply be expressed as

$$K_p(t) = A^2(1-t_1^2)^p(1-t_2^2)^p \qquad (11)$$

The corresponding bivariate Epanechnikov, Biweight and Triweight kernel are

$$K_1(t) = \frac{9}{16}(1-t_1^2)(1-t_2^2) \qquad (12)$$

$$K_2(t) = \frac{225}{256}(1-t_1^2)^2(1-t_2^2)^2 \qquad (13)$$

$$K_3(t) = \frac{1225}{1024}(1-t_1^2)^3(1-t_2^2)^3 \qquad (14)$$

Again, at the limiting case, the bivariate Gaussian kernel is of the form

$$K_\emptyset(t) = \frac{1}{2\pi}\exp\left(-\frac{t_1^2+t_2^2}{2}\right) \qquad (15)$$

The Gaussian kernel is very fundamental in density estimation because it produces smooth density estimates and the mathematical computations can be explicitly expressed. It is also continuously differentiable and possesses derivatives of all order which supported its wide uses in density estimation unlike other members of the beta family.

Performance Measure of Kernel Estimator

The performance of the kernel estimator is dependent on the smoothing parameter and not the kernel function; therefore,

appropriate choices are imperative. One of the optimality criteria functions in kernel density estimation with regards to dimension is the mean integrated squared error with its components given by

$$MISE\left(\hat{f}(x)\right) = \int Var\left(\hat{f}(x)\right)dx + \int Bias^2\left(\hat{f}(x)\right)dx. \quad (16)$$

There is trade-off between the components of the MISE, the bias can be reduced while the variance increases and vice versa, if the magnitude of the smoothing parameter varies. As a result of the importance of the smoothing in performance evaluation of the kernel estimator, appropriate criterion function that regulates the contributions of both components must be employed (Siloko and Siloko, 2019). The approximate form of Equation (16) is the asymptotic mean integrated squared error which produces the integrated variance and the integrated squared bias given by

$$AMISE\left(\hat{f}(x)\right) = \frac{R(K)}{nh_x} + \frac{h_x^4}{4}\mu_2(K)^2 R(f''(x)), \quad (17)$$

where $R(K)$ is roughness of kernel, $\mu_2(K)^2$ is variance of kernel while $R(f''(x)) = \int f''(x)^2 dx$ is the roughness of the unknown density function. The smoothing parameter with the minimum AMISE in Equation (17) in terms of dimension is

$$h_{x-AMISE} = \left[\frac{R(K)}{\mu_2(K)^2 R(f''(x))}\right]^{\left(\frac{1}{4+d}\right)} \times n^{-1/(4+d)}. \quad (18)$$

where d is dimension of the kernel function. The multivariate asymptotic mean integrated squared error with the product kernel estimator is

$$\text{AMISE}\left(\hat{f}(x)\right) = \frac{R(K)^d}{nh_1h_2,\dots,h_d} + \frac{1}{4}h_j^4\mu_2(K)^2 \int tr^2\left(\nabla^2 f(x)\right)dx \quad = \frac{R(K)^d}{nh_1h_2,\dots,h_d} + \frac{1}{4}h_j^4\mu_2(K)^2 R(\nabla^2 f(x)) \quad (19)$$

where $R(K) = \int K^2(x)dx$ is roughness of kernel, $\mu_2(K)^2$ is variance of kernel, $R(\nabla^2 f(x)) = \int tr^2\left(\nabla^2 f(x)\right)dx$ is roughness of the function, tr is trace of a matrix, n is sample size, $h_1, h_2, \dots, h_d$ are the smoothing parameters for each dimension, d is dimension of kernel and $\nabla^2 f(x)$ is Hessian matrix (matrix of second partial derivatives) of the function. The corresponding smoothing parameter with the minimum AMISE value is

$$H_{AMISE} = \left[\frac{dR(K)^d}{\mu_2(K)^2 R(\nabla^2 f(x))}\right]^{\left(\frac{1}{d+4}\right)} \times n^{-\left(\frac{1}{d+4}\right)} \quad (20)$$

As noted above, Equation (18) and Equation (20) depend on the second derivative of the unknown function, that is $R(f''(x))$ and $R(\nabla^2 f(x))$ respectively, therefore it will be difficult to evaluate without the knowledge of the true density function $f(x)$. Several researches and suggestions have been made about the value of $R(f''(x))$ and $R(\nabla^2 f(x))$ for different kernel functions. We obtain the value of the roughness of the function, $R(\nabla^2 f(x))$ with reference to the Epanechnikov, Biweight, and Gaussian functions.

Data Analysis

The statistical properties of the weather variable were obtained with their informative graphical displays also presented in bivariate form with the aid of Mathematica ver.12 software,

www.wolfram.com/Mathematica. This analysis aims at exploring the general distributional properties of the weather observations and obtaining the necessary information for decision making and possibly future predictions. The beta kernel functions employed in this analysis are Epanechnikov, Biweight, and Gaussian kernel functions. The smoothing parameter values that minimize the AMISE of the data were obtained with reference to their respective distributions.

Result and Discussion

The weather data investigated is the ERA-Interim average daily meteorological data that is made up of 365 observations on two variables namely; temperature in degree Celsius and relative humidity in percentages (Dee *et al*., 2011; Ukhurebor and Abiodun, 2018 Ukhurebor and Azi, 2019). The two variables were the daily records of temperature and relative humidity for the year 2018. The analysis of these data addresses the relationship between temperature and relative humidity and their direct impact on the environment. The set of observations were standardized to ensure that there are no variations in their ranges (Ukhurebor and Odesanya, 2019; Siloko *et al*., 2018; Siloko *et al*., 2020b). Standardization of data is of great importance in multivariate analysis particularly in kernel density estimation where visualization of data structure is the primary goal. It should be noted that standardization of data during analysis does not affect the inherent features of the data set. The bivariate kernel estimates of the weather variables using the bivariate product kernel are in Figure 1, Figure 2 and Figure 3 using Epanechnikov, Biweight, and Gaussian kernel functions. The smoothing parameter and the asymptotic mean integrated squared error (AMISE) values are in Table 1. The Epanechnikov being the optimal kernel produce the smallest value of the AMISE amongst the three kernel functions examined. Again, the smoothing parameter increases as the power of the kernel increase as seen in Table 1 while the Normal kernel produced the largest value of the AMISE. In kernel method, performance is usually attributed to the ability of the kernel function in producing the smallest AMISE using the smoothing parameter (Jarnicka, 2009; Siloko *et al*., 2017). The results in Table 1 show that the Epanechnikov kernel outperformed the Biweight and Gaussian kernels due to its production of the smallest AMISE value and that supports its optimality with respect to the AMISE as a criterion function.

The bivariate kernel estimates of the weather variables clearly show that the data are multimodal which indicates that the variables are inversely related. The multimodality of the kernel estimates of the observations implies the direct numerous effects of the variables on human activities and the environment. The Gaussian kernel produces smooth kernel estimates as seen in Figure 3 but despite the noise in the Epanechnikov and Biweight estimates as noticed in Figure 1 and Figure 2, the clear effect of temperature and relative humidity is displayed statistically. The effects of the interaction of temperature and relative humidity investigated are with high probability value as seen in the kernel estimates. The probability value ranges between 0 and 0.14 which means any human action that can be affected by these variables in the environment if not immediately controlled will result in disaster to mankind.
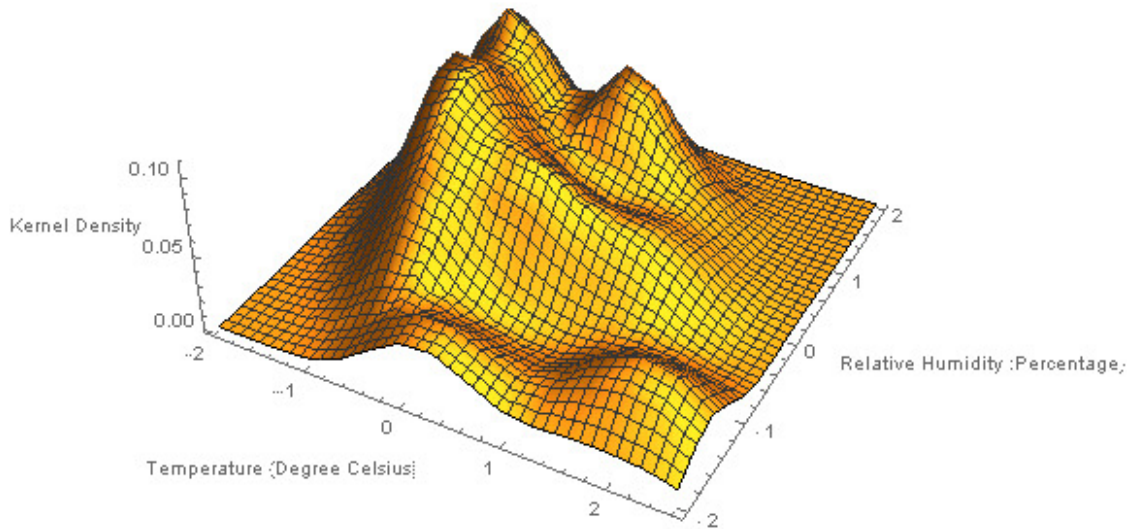
Fig. 1a: Surface plot of Epanechnikov bivariate kernel density estimate of weather data
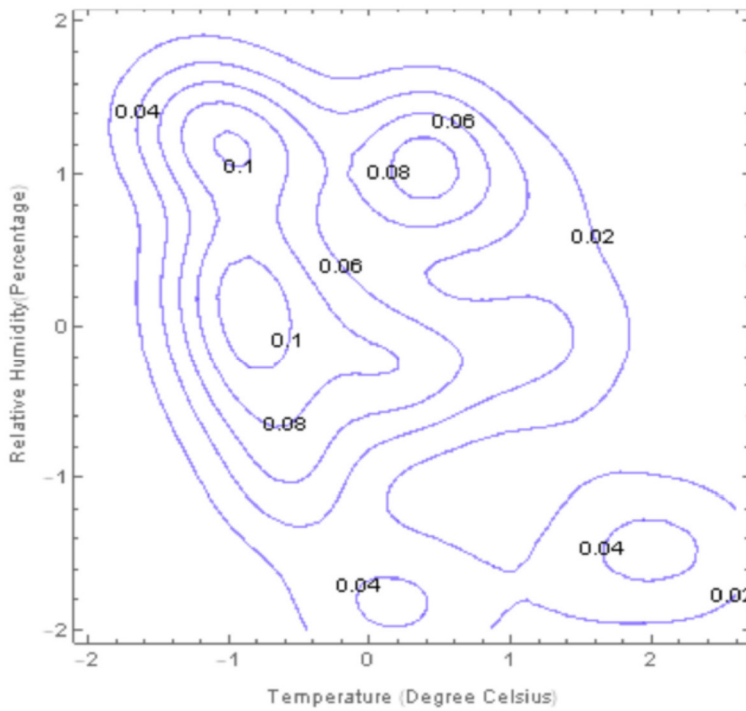

Fig. 1b: Contour plot of Epanechnikov bivariate kernel density estimate of weather data
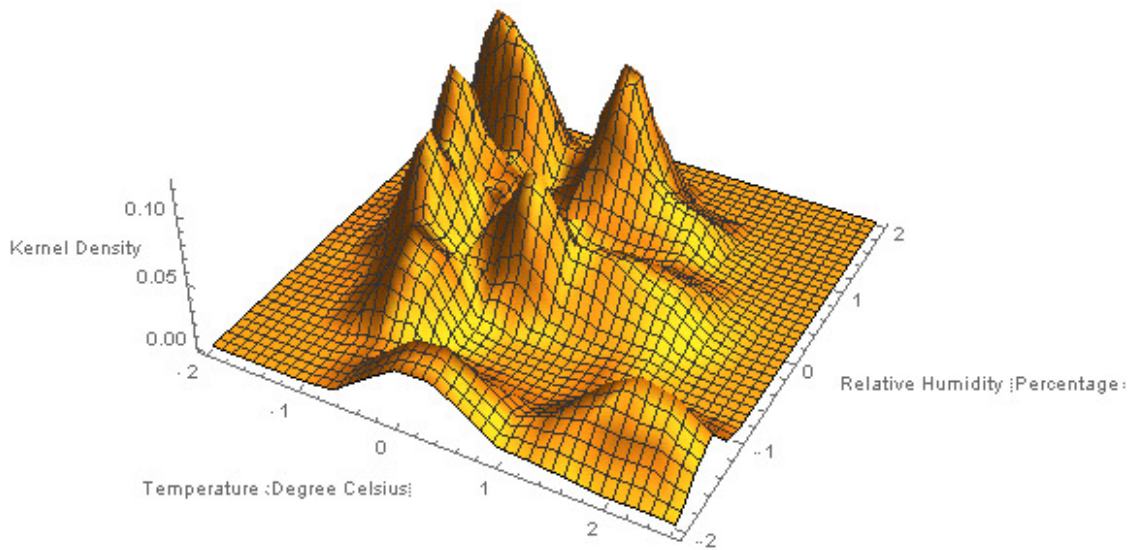
Fig. 2a: Surface plot of Biweight bivariate kernel density estimate of weather data
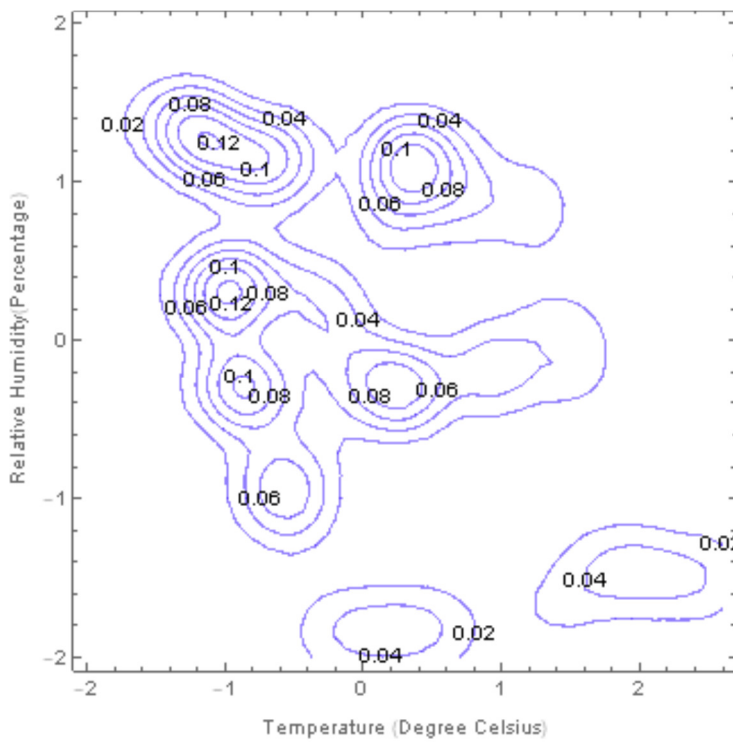

Fig. 2b: Contour plot of Biweight bivariate kernel density estimate of weather data
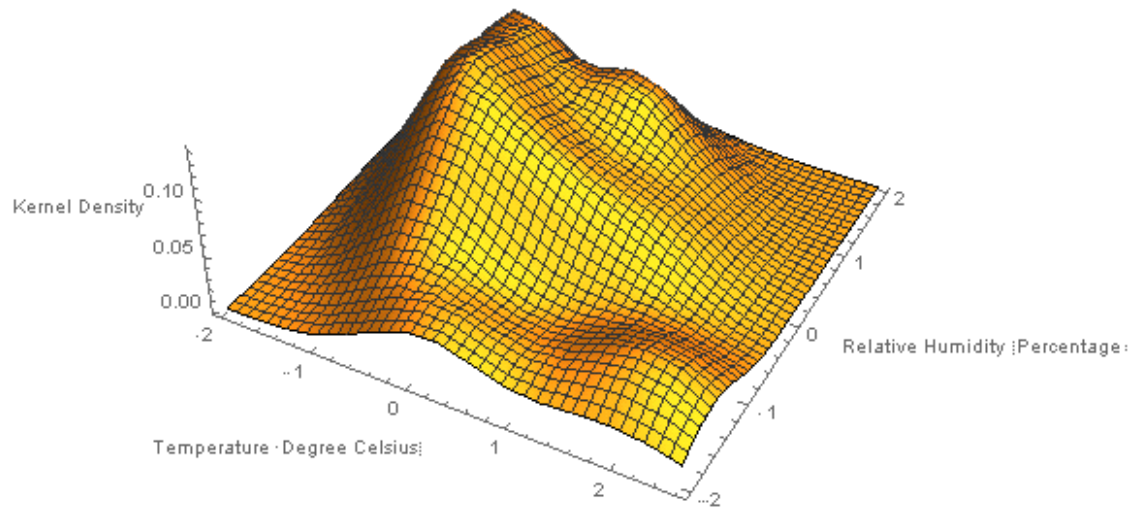
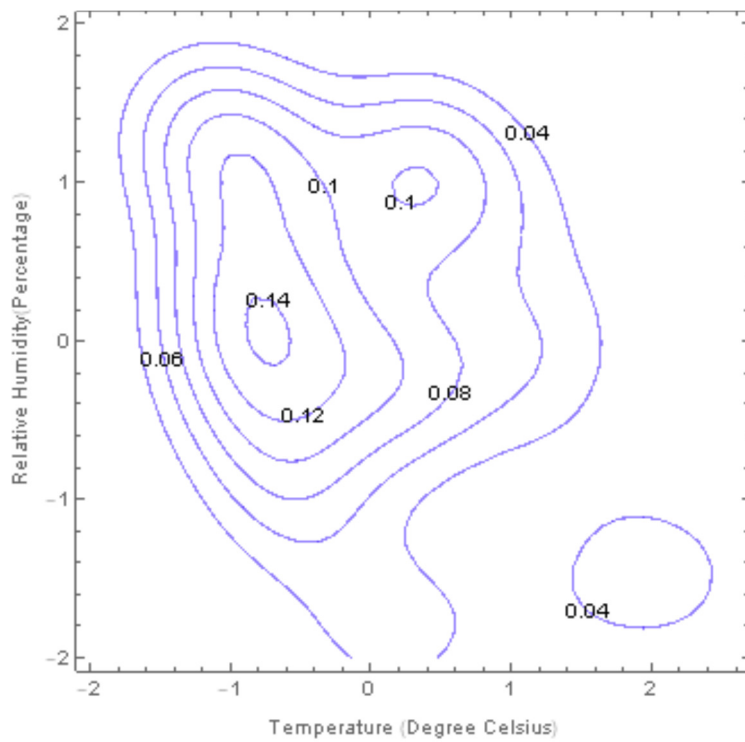Fig. 3a: Surface plot of Normal bivariate kernel density estimate of weather data

Fig. 3b: Contour plot of Normal bivariate kernel density of weather data

Table1: Kernel Functions, Smoothing Parameters, Variance, $Bias^2$ and AMISE of the Data

Kernel Function	h_x	h_y	Variance	$Bias^2$	AMISE
Epanechnikov	0.486475	0.503703	0.00046124	0.00016110	0.00062234
Biweight	0.451965	0.451834	0.00111163	0.00013969	0.00125132
Gaussian	0.449998	0.457424	0.00105918	0.00105931	0.00211849

The inverse relationship between the studied observations has serious effect on man and its environment because an increase in relative humidity will result in more water in the atmosphere that could circulate malodorous molecules from bacteria related sources to human. The emission of dangerous chemicals into the environment is one of the ways by which human activities have affected its environment adversely because such activities could result in increase in temperature and reduce the quality of air that ultimately leads to death of plants and animals. Cardiovascular diseases that could lead to death are also connected with the observations examined; hence the multimodality of the kernel estimates. Therefore, maintenance of the environment should be our priority so that the effects of increase humidity and heat due to dirtiness can be avoided.

Conclusion

Temperature and relative humidity are inversely related and this inverse relationship as investigated using real life data shows its devastating effect on the environment if neglected without urgent control measure. The probability of their interaction is high using the statistical tools of nonparametric density estimation. Activities that could result in depletion of the Ozone layer should be avoided and environmentalists should as a matter of urgency sensitize the citizenry on the importance of urgent need of maintenance of the environment. Government at all levels in every nation should come up with stringent policies that regulate the activities of individuals and industries towards the maintenance of the environment to reduce the effects of climatic fluctuations.

References

Adejuwon, S.A. (2004). Impacts of climate variability and climate change on crop yield in Nigeria. A lead paper presented at the workshop on assessment of impacts and adaptation to climate change (AIACC), at Obafemi Awolowo University, Ile-Ife, Nigeria, 20–21 September, 2004.

Bobadoye, A.O., Ukhurebor, K.E., Siloko, I.U., Nwankwo, W. and Odesanya, I. (2020). Estimation of some thermodynamic meteorological variables from measured meteorological Data. *Ethiopian Journal of Environmental Studies and Management*, 13(5): 620–632.

Brown, M.E., Antle, J.M., Backlund, P., Carr, E.R., Easterling, W.E., Walsh, M.K., Ammann, C., Attavanich, W., Barrett, C.B., Bellemare, M.F., Dancheck, V., Funk, C., Grace, K., Ingram, J.S.I., Jiang, H., Maletta, H., Mata, T., Murray, A., Ngugi, M., Ojima, D., O'Neill, B. and Tebaldi, C. (2015). Climate Change, Global Food Security, and the U.S. Food System. 146 Pages.

Dee, D.P., Uppala, S.M., Simmons, A.J., Berrisford, P., Poli, P., Kobayashi, S., Andrae, U., Balmaseda, M. A., Balsamo, G., Bauer, P., Bechtold,

P., Beljaars, A.C.M., van de Berg, L., Bidlot, J., Bormann, N., Delsol, C., Dragani, R., Fuentes, M., Geer, A. J., Haimberger, L., Healy, S.B., Hersbach, H., Hólm, E.V., Isaksen, L., Kållberg, P., Köhler, M., Matricardi, M., McNally, A. P., Monge-Sanz, B.M., Morcrette, J. -J., Park, B. -K., Peubey, C., de Rosnay, P., Tavolato, C., Thépaut, J. -N. and Vitart, F. (2011). The ERA-Interim reanalysis: configuration and performance of the data assimilation system. *Quarterly Journal of the Royal Meteorological Society*, 137: 553–597.

Hansen, B.E. (2019). Econometrics. Department of Economics, University of Wisconsin.

Jarnicka, J. (2009). Multivariate kernel density estimation with a parametric support. *Opuscula Mathematica*, 29(1): 41–45.

Lewis, P., Monem, M. A. and Impiglia, A. (2018). Impacts of climate change on farming systems and livelihoods in the near east and North Africa with a special focus on small-scale family farming. Cairo, FOA. 92pp.

Niggol. S. and Mendelsohn, R. (2008). Animal Husbandry in Africa: Climate impacts and adaptations. *AfJARE*, 2(1): 66.

Nwankwo, W. and Ukhurebor, K. E. (2019). An x-ray of connectivity between climate change and particulate pollutions. *Journal of Advanced Research in Dynamical and Control Systems, Special Issue*, 11(8): 3002–3011.

Parzen, E. (1962). On the estimation of a probability density function and the mode. Annals of Mathematical Statistics, 33, 1065–1076.

Raykar, V.C., Duraiswami, R. and Zhao, L.H. (2015). Fast computation of kernel estimators. *Journal of Computational and Graphical Statistics*, 19(1): 205–220.

Rosenblatt, M. (1956). Remarks on some nonparametric estimates of a density function. *Annals of Mathematical Statistics*, 27: 832–837.

Sain, R.S., Baggerly, A.K. and Scott, D.W. (1994). Cross-validation of multivariate densities. *Journal of American Statistical Association*, 89: 807–817.

Scott, D.W. (2015). Multivariate density estimation: Theory, practice and visualisation. Second Edition, Wiley, New Jersey.

Siloko, I.U. and Siloko, E.A. (2019). Density estimation and data analysis using the kernel method. *Proceedings of 56th Annual National Conference of Mathematical Association of Nigeria, 218–226*.

Siloko, I.U., Ikpotokin, O. and Siloko, E.A. (2019a). A key note on performance of smoothing parameterizations in kernel density estimation. *Tanzania Journal of Science*, 45(1): 1–8.

Siloko, I.U., Ikpotokin, O. and Siloko, E.A. (2019b). On hybridizations of fourth order kernel of the beta polynomial family. *Pakistan Journal of Statistics and Operations Research*, 15(3): 819–829.

Siloko, I.U., Ishiekwene, C.C. and Oyegue, F.O. (2017). The Effect of Smoothing Parameter in Kernels Aggregation. *Canadian Journal of Pure and Applied Sciences*, 11(2): 4255–4262.

Siloko, I.U., Ishiekwene, C.C. and Oyegue, F.O. (2018). New gradient

methods for bandwidth selection in bivariate kernel density estimation. *Mathematics and Statistics*, 6(1): 1–8.

Siloko, I.U., Nwankwo, W. and Siloko, E. A. (2020a). A new family of kernels from the beta polynomial kernels with applications in density estimation. *International Journal of Advances in Intelligent Informatics*, 6(3): 235–245.

Siloko, I.U., Nwankwo, W. and Umezuruike, C. (2020b). A discourse on smoothing parameterizations using hypothetical dataset. *Journal of Applied Sciences, Information, and Computing*, 1(1): 80–88.

Silverman, B.W. (2018). Density estimation for statistics and data analysis. Routledge, New York.

Smith, M. (2000). The application of climatic data for planning and management of sustainable rainfall and irrigated crop production. *Agricultural and Forest Meteorology*, 103: 99–108.

Tompkins, A.M. (2003). Impact of temperature and humidity variability on cloud cover assessed using aircraft data. *Quarterly Journal of the Royal Meteorological Society,* 129: 2151–2170.

Ukhurebor, K.E. and Abiodun, I.C. (2018). Variation in annual rainfall data of forty years (1978-2017) for South-South, Nigeria. *Journal of Applied Sciences and Environmental Management*, 22(4): 511–518.

Ukhurebor, K.E. and Azi, S.O. (2019). Review of methodology to obtain parameters for radio wave propagation at low altitudes from meteorological data: New results for Auchi area in Edo State, Nigeria. *Journal of King Saud University–Science*, 31(4): 1445–1451.

Ukhurebor, K.E. and Odesanya I. (2019). Relationship between meteorological variables and effective earth radius factor over Auchi, Edo State, South-South, Nigeria. *Covenant Journal of Physical and Life Sciences*, 7(1): 1–10.